



HINDU COLLEGE, GUNTUR – 522 003

(Re-accredited by NAAC as Grade 'A' with CGPA 3.07)

Main Road, Opp. Sri Venkateswara Vignan Mandir, Guntur

DEPARTMENT OF MATHEMATICS

DR. S.V.S. GIRIJA, Professor of Mathematics

Y. SREEKANTH, Lecturer in Mathematics

STUDENT CENTRIC ACTIVITIES – 2021-22

MAGICAL JOURNEY ON MOBIUS STRIP

Problem Solving Session 1

Problem	: Magical Journey on Mobius Strip
Date and Time	: 17-02-2022 12.30 to 1.20
Venue	: N5
Co-ordinator	: Dr. S.V.S. Girija
No.of Staff Members Involved: One (Y. Sreekanth)	
No.of Students Involved	: 34 II B.Sc. MPC , MPCs, MSCs
Objective of the event	: To make students well versed with new concepts and solving problems
Plan and Execution	: Problem is given to the Students to demonstrate the solution and applications of the concepts.
Outcome of the Event	: Students studied the concepts and explained in such a way that everyone is able to understand the mathematical concepts and applications of Mobius Strip. They demonstrated with models.
Evidences	: Material and Pictures



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In mathematics, a **Möbius strip**, **Möbius band**, or **Möbius loop** is a surface that can be formed by attaching the ends of a strip of paper together with a half-twist. As a mathematical object, it was discovered by Johann Benedict Listing and August Ferdinand Möbius in 1858, but it had already appeared in Roman mosaics from the third century CE. The Möbius strip is a non-orientable surface, meaning that within it one cannot consistently distinguish clockwise from counterclockwise turns. Every non-orientable surface contains a Möbius strip.

As an abstract topological space, the Möbius strip can be embedded into three-dimensional Euclidean space in many different ways: a clockwise half-twist is different from a counterclockwise half-twist, and it can also be embedded with odd numbers of twists greater than one, or with a knotted centerline. Any two embeddings with the same knot for the centerline and the same number and direction of twists are topologically equivalent. All of these embeddings have only one side, but when embedded in other spaces, the Möbius strip may have two sides. It has only a single boundary curve.

Several geometric constructions of the Möbius strip provide it with additional structure. It can be swept as a ruled surface by a line segment rotating in a rotating plane, with or without self-crossings. A thin paper strip with its ends joined to form a Möbius strip can bend smoothly as a developable surface or be folded flat; the flattened Möbius strips include the trihexaflexagon. The Sudanese Möbius strip is a minimal surface in a hypersphere, and the Meeks Möbius strip is a self-intersecting minimal surface in ordinary Euclidean space. Both the Sudanese Möbius



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strip and another self-intersecting Möbius strip, the cross-cap, have a circular boundary. A Möbius strip without its boundary, called an open Möbius strip, can form surfaces of constant curvature. Certain highly-symmetric spaces whose points represent lines in the plane have the shape of a Möbius strip.

The many applications of Möbius strips include mechanical belts that wear evenly on both sides, dual-track roller coasters whose carriages alternate between the two tracks, and world maps printed so that antipodes appear opposite each other. Möbius strips appear in molecules and devices with novel electrical and electromechanical properties, and have been used to prove impossibility results in social choice theory. In popular culture, Möbius strips appear in artworks by M. C. Escher, Max Bill, and others, and in the design of the recycling symbol. Many architectural concepts have been inspired by the Möbius strip, including the building design for the NASCAR Hall of Fame. Performers including Harry Blackstone Sr. and Thomas Nelson Downs have based stage magic tricks on the properties of the Möbius strip. The canons of J. S. Bach have been analyzed using Möbius strips. Many works of speculative fiction feature Möbius strips; more generally, a plot structure based on the Möbius strip, of events that repeat with a twist, is common in fiction.



Mosaic from ancient [Sentinum](#) depicting [Aion](#) holding a Möbius strip



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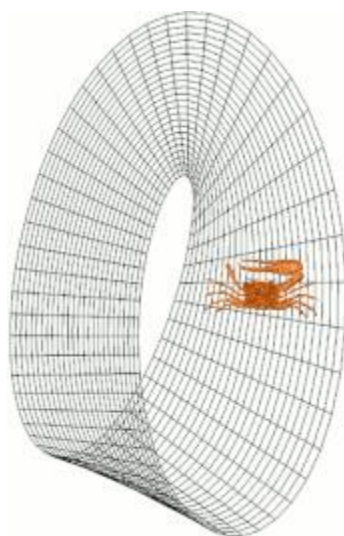
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PROPERTIES:



A 2D object traversing once around the Möbius strip returns in mirrored form

The Möbius strip has several curious properties. It is a **non-orientable surface**: if an asymmetric two-dimensional object slides one time around the strip, it returns to its starting position as its mirror image. In particular, a curved arrow pointing clockwise ($\curvearrowright$) would return as an arrow pointing counterclockwise ($\curvearrowleft$), implying that, within the Möbius strip, it is impossible to consistently define what it means to be clockwise or counterclockwise.

A path along the edge of a Möbius strip, traced until it returns to its starting point on the edge, includes all boundary points of the Möbius strip in a single continuous curve. For a Möbius



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strip formed by gluing and twisting a rectangle, it has twice the length of the centerline of the strip.

In this sense, the Möbius strip is different from an untwisted ring and like a circular disk in having only one boundary.

The Möbius strip can be cut into six mutually-adjacent regions, showing that maps on the surface of the Möbius strip can sometimes require six colors, in contrast to the [four color theorem](#) for the plane.^[19] Six colors are always enough. This result is part of the [Ringel–Youngs theorem](#), which states how many colors each topological surface needs.^[20] The edges and vertices of these six regions form [Tietze's graph](#), which is a [dual graph](#) on this surface for the six-vertex [complete graph](#) but cannot be [drawn without crossings on a plane](#). Another family of graphs that can be [embedded](#) on the Möbius strip, but not in the plane, are the [Möbius ladders](#), the boundaries of subdivisions of the Möbius strip into rectangles meeting end-to-end.^[21] These include the utility graph, a six-vertex [complete bipartite graph](#) whose embedding into the Möbius strip shows that, unlike in the plane, the [three utilities problem](#) can be solved on a transparent Möbius strip.^[22] The [Euler characteristic](#) of the Möbius strip is [zero](#), meaning

that for any subdivision of the strip by vertices and edges into regions, the numbers

, and of vertices, edges, and regions satisfy . For instance, Tietze's graph

has vertices, edges, and regions.

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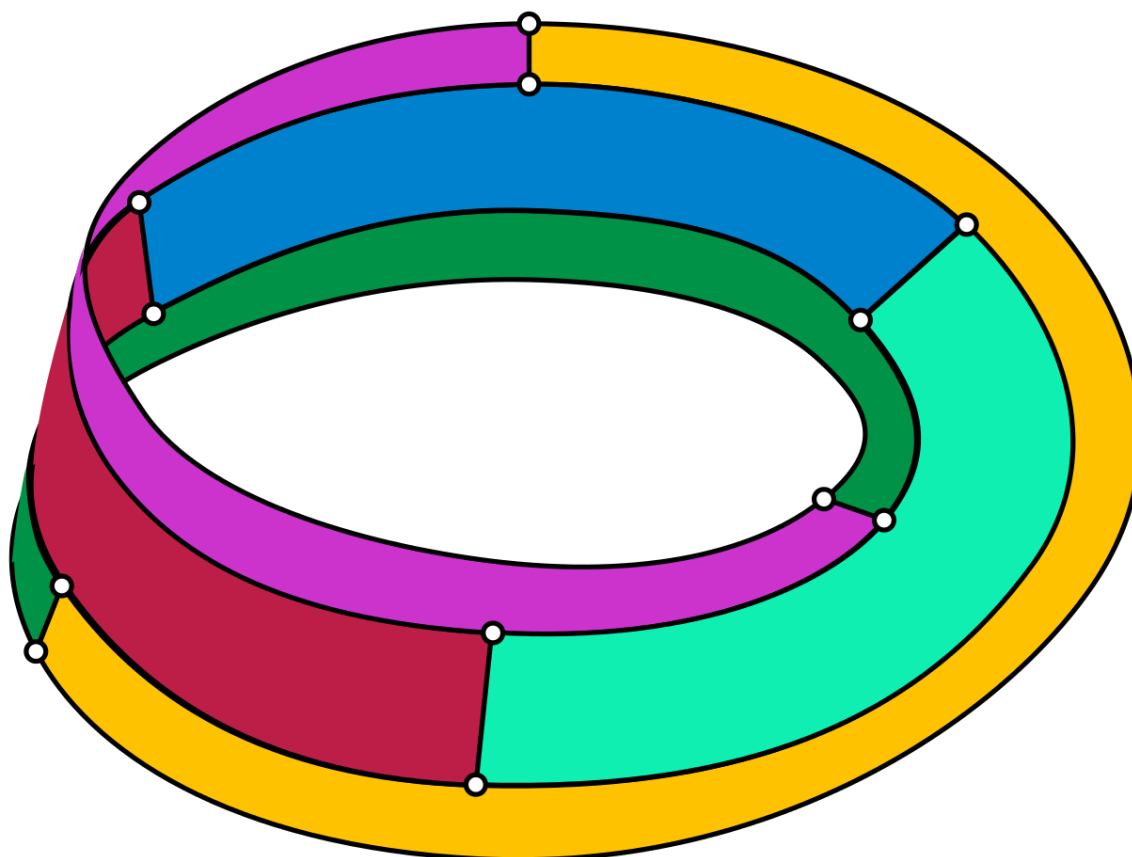


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2009-07-20



A Möbius strip swept out by a rotating line segment in a rotating plane

Sweeping a line segment[\[edit\]](#)

A Möbius strip swept out by a rotating line segment in a rotating plane.

[Plücker's conoid](#) swept out by a different motion of a line segment

One way to embed the Möbius strip in three-dimensional Euclidean space is to sweep it out by a line segment rotating in a plane, which in turn rotates around one of its lines.^{[\[23\]](#)} For the swept surface to meet up with itself after a half-twist, the line segment should rotate around its center



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at half the angular velocity of the plane's rotation. This can be described as a [parametric surface](#) defined by equations for the [Cartesian coordinates](#) of its points,

$$\begin{aligned}x(u, v) &= \left(1 + \frac{v}{2} \cos \frac{u}{2}\right) \cos u \\y(u, v) &= \left(1 + \frac{v}{2} \cos \frac{u}{2}\right) \sin u \\z(u, v) &= \frac{v}{2} \sin \frac{u}{2}\end{aligned}$$

for $0 \leq u < 2\pi$ and $-1 \leq v \leq 1$,

where one parameter u describes the rotation angle of the plane around its central axis and the other parameter v describes the position of a point along the rotating line segment. This produces a Möbius strip of width 1, whose center circle has radius 1, lies in the XY-plane and is centered at (0,0,0). The same method can produce Möbius strips with any odd number of half-twists, by rotating the segment more quickly in its plane. The rotating segment sweeps out a circular disk in the plane that it rotates within, and the Möbius strip that it generates forms a slice through the [solid torus](#) swept out by this disk. Because of the one-sidedness of this slice, the sliced torus remains connected.



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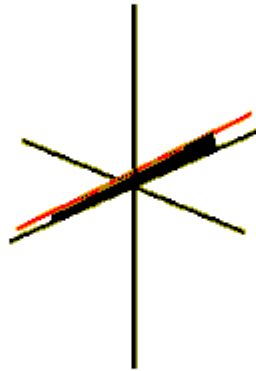
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[Plücker's conoid](#) swept out by a different motion of a line segment





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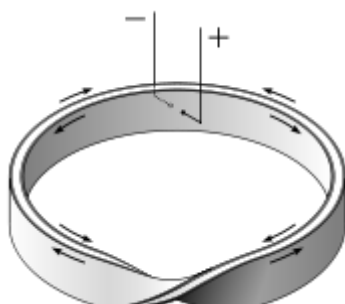
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Lawson's Klein bottle is a self-crossing [minimal surface](#) in the [unit hypersphere](#) of 4-dimensional space, the set of points of the form

$$(\cos \theta \cos \phi, \sin \theta \cos \phi, \cos 2\theta \sin \phi, \sin 2\theta \sin \phi),$$
$$\text{for } 0 \leq \theta < \pi, 0 \leq \phi < 2\pi$$

for [\[51\]](#) Half of this Klein bottle, the subset with $0 \leq \theta < \pi$, gives a Möbius strip embedded in the hypersphere as a minimal surface with a [great circle](#) as its boundary. [\[52\]](#) This embedding is sometimes called the "Sudanese Möbius strip" after topologists Sue Goodman and Daniel Asimov, who discovered it in the 1970s.



Electrical flow in a [Möbius resistor](#)

APPLICATIONS

- Mechanical belts that wear evenly on both sides
- Dual-track roller coasters whose carriages alternate between the two tracks
- World maps printed so that antipodes appear opposite each other.
- Möbius strips appear in molecules and devices with novel electrical and electromechanical properties
- Have been used to prove impossibility results in social choice theory.



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- Many architectural concepts have been inspired by the Möbius strip, including the building design for the NASCAR Hall of Fame.
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